On the $K^{\pm}$–meson production from the quark–gluon plasma phase in ultra–relativistic heavy–ion collisions

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Abstract

An abundance of the strangeness that can be induced in a thermalized quark–gluon plasma (QGP) is considered as a signal of the QGP phase appearing in the intermediate state of ultra–relativistic heavy–ion collisions. As a quantitative characteristic of this signal we take the ratio $R_{K^+K^-} = N_{K^+}/N_{K^-}$ of the multiplicities of the production of $K^{\pm}$ mesons. This ratio is evaluated for a thermalized QGP phase of QCD and for the quark–gluon system escaped from the QGP phase. For a thermalized QGP phase the ratio $R_{K^+K^-}$ has been found as a smooth function of a 3–momentum of the $K^{\pm}$ mesons and a temperature ranging the values from the region $160\,\mathrm{MeV} < T < 200\,\mathrm{MeV}$. We show that at the temperature $T = 175\,\mathrm{MeV}$ our prediction for the ratio $R_{K^+K^-}(q, T = 175) = 1.80^{+0.04}_{-0.18}$ agrees good with the experimental data of NA49 and NA44 Collaborations on central ultra–relativistic Pb–Pb collisions at 158 GeV per nucleon: $R^{\exp}_{K^+K^-} = 1.80 \pm 0.10$. For the ratio of the K^+ and π^+ multiplicities we have obtained the value $R_{K^+\pi^+}(q, T = 175) = 0.134 \pm 0.014$ agreeing good with the experimental data of NA35 Collaboration on the nucleus–nucleus collisions at $200\,\mathrm{GeV}$ per nucleon $R^{\exp}_{K^+\pi^+} = 0.137 \pm 0.008$.

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1 Introduction

Nowadays there is a consensus that QCD gives a satisfactory description of strong interactions of hadrons. The important question still left concerns the properties of the QCD vacuum. One of the approaches to the exploration of the properties of the QCD vacuum is to investigating the excited vacuum states at high densitities and temperature. The quark–gluon plasma (QGP) phase of QCD [1,2] is just the excited QCD vacuum in which quarks, anti–quarks and gluons being at the deconfined phase collide frequently each other. There is a belief [2] that the QGP phase of the quark–gluon system can be realized in ultra–relativistic heavy–ion collision ($E_{\rm cms}$/nucleon $\gg 1\,{\rm GeV}$) experiments.

There are two main approaches to the description of the QGP phase – the dynamical [3] and the thermal [4] ones. The dynamical approach employs the methods related to the relativistic kinetic equations supplemented by semi–hard interactions at very high energies. The thermal approach is based on the ideal gas assumption supposing a thermalization of the QGP after some initial time τ_0. Finally, in both approaches the QGP phase evolutes to the hadronic phase.

The difficulties arising from the recognition of the QGP, induced in ultra–relativistic heavy–ion collisions, through a hadronization concern the following. The hadrons can also be produced in heavy–ion collisions by the quark–gluon system escaped from the QGP phase. Therefore, one needs to have distinct criteria allowing to distinguish the hadrons produced by the QGP phase from the hadrons procreated by the quark–gluon system escaped from the intermediate QGP phase.

As has been suggested in Refs.[5,6] in a thermalized QGP one can expect an abundance of the production of strange hadrons K, Λ, Ξ and so on. Such an abundance can serve as a criterion for the formation of the QGP [5,6]. The arguments for the enhancement of the strange hadron production are the following. At very high energies of heavy–ion collisions the quark–gluon system is composed from highly relativistic and very dense quarks, anti–quarks and gluons. By virtue of the asymptotic freedom the particles are almost at liberty and due to high density collide themselves frequently that leads to an equilibrium state. If to consider such a state as a thermalized QGP phase of QCD, the probabilities of light massless quarks $n_q(\vec{p}\,)$ and light massless anti–quarks $n_{\bar{q}}(\vec{p}\,)$, where $q = u$ or d, to have a momentum p at a temperature T, can be described by the Fermi–Dirac distribution functions [1,4,7]:

$$n_q(\vec{p}\,) = \frac{1}{e^{-\nu(T) + p/T} + 1} \quad , \quad n_{\bar{q}}(\vec{p}\,) = \frac{1}{e^{\nu(T) + p/T} + 1}, \tag{1.1}$$

where a temperature T is measured in MeV, $\nu(T) = \mu(T)/T$, $\mu(T)$ is a chemical potential of the light massless quarks $q = u, d$, depending on a temperature T [7]. A chemical potential of light anti–quarks amounts to $-\mu(T)$. A positively defined $\mu(T)$ provides an abundance of light quarks with respect to light anti–quarks for a thermalized state [1,4]. A chemical potential $\mu(T)$ is a phenomenological parameter of the approach which we would fix below.

The probability for gluons to have a momentum $\vec{p}$ at a temperature T is given by the Bose–Einstein distribution function

$$n_g(\vec{p}\,) = \frac{1}{e^{p/T} - 1}. \tag{1.2}$$

Since a strangeness of the colliding heavy–ions amounts to zero, the densities of strange quarks and anti–quarks should be equal. The former implies a zero–value of a chemical potential $\mu_s = \mu_{\bar{s}} = 0$. In this case the probabilities of strange quarks and anti–quarks can be given by

$$n_s(\vec{p}\,) = n_{\bar{s}}(\vec{p}\,) = \frac{1}{e^{\sqrt{\vec{p}^{\,2} + m^2_s}/T} + 1}, \tag{1.3}$$

where $m_s = 135\,\mathrm{MeV}$ [8] is the mass of the strange quark and anti–quark. The value of the current s–quark mass $m_s = 135\,\mathrm{MeV}$ has been successfully applied to the calculation of chiral corrections to amplitudes of low–energy interactions, form factors and mass spectra of low–lying hadrons [9] and charmed heavy–light mesons [10]. Unlike the massless anti–quarks $\bar{u}$ and $\bar{d}$ for which the suppression is caused by a chemical potential $\mu(T)$, the strange quarks and anti–quarks are suppressed by virtue of the non–zero mass m_s.

The multiplicities of the production of K^+ and K^- mesons N_{K^+} and N_{K^-}, correspondingly, we describe in the simple coalescence approach [4,6]. In this case the multiplicities can be related to the probabilities of the quarks and anti–quarks as follows:

$$\begin{aligned} N_{K^+}(\vec{q},T) &= \;<n_u(\vec{p}-\vec{q})\,n_{\bar{s}}(\vec{p})> \;= \\ &= N_C V_K \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{e^{-\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\,\frac{1}{e^{\sqrt{\vec{p}^{\,2}+m^2_s}/T}+1}, \\ N_{K^-}(\vec{q},T) &= \;<n_{\bar{u}}(\vec{p}-\vec{q})\,n_s(\vec{p})> \;= \\ &= N_C V_K \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{e^{\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\,\frac{1}{e^{\sqrt{\vec{p}^{\,2}+m^2_s}/T}+1}, \end{aligned} \tag{1.4}$$

where $\vec{q}$ is a 3–momentum of the $K^{\pm}$ mesons, $N_C = 3$ is the number of quark colour degrees of freedom. Then, V_K is a parameter of a coalescence model [4,6] having dimension of volume and being to some extent an intrinsic characteristic of spatial distribution of the K meson. We suggest to determine V_K in terms of the K–meson parameters as follows. Due to the uncertainty relations the K–meson should be localized to the region proportional to the inverse power of a 3–momentum. For the thermalized K–meson system this should be a thermal momentum. In the case of the Maxwell–Boltzmann K–meson gas the thermal momentum is proportional to $\sqrt{M_K}$, where $M_K = 500\,\mathrm{MeV}$ is the K–meson mass. Another important intrinsic parameter of pseudoscalar mesons is the leptonic coupling constant F_P, $F_K = 160\,\mathrm{MeV}$ for the K mesons. Thus, from dimensional consideration we suggest to set $V_K = C/(F_K M_K)^{3/2}$, where C is a dimensionless parameter of the approach equal for all pseudoscalar mesons. Of course, such a determination of V_K is not so much rigorous, but it can be useful as a working hypothesis providing a good agreement with the experimental data.

The multiplicity of the π^+–meson production we define in an analogous way:

$$\begin{aligned} N_{\pi^+}(\vec{q},T) &= \;<n_u(\vec{p}-\vec{q})\,n_{\bar{d}}(\vec{p})> \;= \\ &= N_C V_\pi \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{e^{-\nu(T)+|\vec{p}-\vec{q}\,|/T}+1}\,\frac{1}{e^{\nu(T)+p/T}+1}, \end{aligned} \tag{1.5}$$

where $V_\pi = C/(F_\pi M_\pi)^{3/2}$, $F_\pi = 131\,\mathrm{MeV}$ and $M_\pi = 140\,\mathrm{MeV}$ are the leptonic coupling constant and the mass of pions.

We should emphasize that other dimensional parameters of K and π mesons like charge radii r_{K^+} and r_{π^+} cannot be considered as intrinsic parameters of these mesons, since they are functions of F_K and F_π. For example, in the Vector Dominance approach the charge radii can be expressed in terms of the masses of the ρ and K^* vector mesons M_ρ and M_{K^*}. According to KSFR relations imposed by chiral symmetry [9] these masses are proportional to F_π and F_K, respectively: $M_\rho = g_\rho F_\pi \simeq 790\,\mathrm{MeV}$ and $M_{K^*} = g_\rho F_K \simeq 960\,\mathrm{MeV}$, where $g_\rho \simeq 6$ is the coupling constant of the $\rho\pi\pi$ interaction. The theoretical values of the masses of the ρ and K^* mesons predicted by the KSFR relations agree with experimental values $M_\rho = 770\,\mathrm{MeV}$ and $M_{K^*} = 892\,\mathrm{MeV}$ within accuracies better than 3% and 8%, respectively.

The main goal of our paper is to calculating the ratios of the multiplicities

$$R_{K^+K^-}(q,T) \;=\; \frac{N_{K^+}(\vec{q},T)}{N_{K^-}(\vec{q},T)},$$

$$R_{K^+\pi^+}(q,T) \;=\; \frac{N_{K^+}(\vec{q},T)}{N_{\pi^+}(\vec{q},T)} \tag{1.6}$$

with the minimum number of input parameters. Since the main input parameter of the thermalized approach to the QGP is a chemical potential $\mu(T)$ of the light quarks and anti–quarks, we would focus on the possibility to fix it.

For this aim we suggest to treat a heavy ion as a degenerated Fermi gas. Converting formally all nucleon degrees of freedom into quark degrees of freedom we end up with a degenerated Fermi gas of quarks or differently a degenerated quark–gluon system, where all gluon and anti–quark degrees of freedom are died out. Heating this quark–gluon system up to the temperature T and demanding the conservation of the baryon number, that corresponds to the conservation of the baryon number density in the fixed spatial volume of the ion, we fix unambiguously a temperature dependence of a chemical potential of light quarks and anti–quarks $\mu(T)$ dropping very swiftly at high temperatures, and the value $\mu(0) = \mu_0 = 250\,\mathrm{MeV}$. A steep falloff of a chemical potential with a temperature implies that at high temperatures the ratio behaves like $R_{K^+K^-}(q,T) \to 1$. In reality, the limit $R_{K^+K^-}(q,T) \simeq 1$ can be reached already at $T \ge \mu_0$, where $\mu_0 \simeq 250\,\mathrm{MeV}$ [1]. However, for intermediate temperatures $T = 160 \div 200\,\mathrm{MeV}$ as it is estimated in Sect. 6 the ratio $R_{K^+K^-}(q,T)$ ranges values from the region $2.14 \div 1.48$. This implies that the experimental analysis of the ratio of the multiplicities of the $K^\pm$–meson production can be a good criterion for the signal of the QGP phase. Indeed, for the $K^\pm$ mesons produced by the quark–gluon system escaped from the QGP phase the ratio N_{K^+}/N_{K^-} is expected to be of order of unity.

The paper is organized as follows. In Section 2 we determine a chemical potential $\mu(T)$. In Section 3 we discuss in short a thermalized QGP. In Section 4 we analyse the multiplicities of the $K^\pm$–meson production and give the analytical formulas for the ratio $R_{K^+K^-}(q,T)$ as a function of 3–momenta of the $K^\pm$ mesons and a temperature T. In Section 5 we analyse the multiplicities of the $\pi^\pm$–meson production and give the analytical formulas for the ratio $R_{K^+\pi^+}(q,T)$ as a function of 3–momenta of K^+ and π^+ mesons and a temperature T. In Section 6 we make the numerical analysis of the analytical formulas derived in Section 4 and Section 5. We show that the ratios depend smoothly on both 3–momenta of $K^\pm$ and π^+ mesons and the temperature ranging values from the region $160\,\mathrm{MeV} \le T \le 200\,\mathrm{MeV}$. We find that for the temperatures $T = 175\,\mathrm{MeV}$ our predictions agree reasonably well with the experimental data by NA49 and NA44 Collaborations on the central relativistic Pb–Pb collisions at 158 GeV per nucleon and the data by NA35 Collaboration for proton–nucleus and nucleus–nucleus collisions at 200 GeV per nucleon. In the Conclusion we discuss the obtained results.

2 Chemical potential of light quarks and anti–quarks

We suppose that a chemical potential $\mu(T)$, a phenomenological parameter of the description of the QGP state as a thermalized quark–gluon system at a temperature T, is an intrinsic characteristic of a thermalized quark–gluon system. Thereby, if the QGP is an excited state of the QCD vacuum, so a chemical potential should exist not only for ultra–relativistic heavy–ion collisions. Quark distribution functions of a thermalized quark–gluon system at a temperature T should be characterized by a chemical potential $\mu(T)$ for any external state and any external conditions. Since any state of a thermalized system is closely related to external conditions, in order to obtain $\mu(T)$ we need only to specify the external conditions of a thermalized quark–gluon system the convenient for the determination of $\mu(T)$.

Let the external conditions of a thermalized quark–gluon system be caused by the state of the nuclear system, which is a heavy ion with a baryon number A. In the Fermi–gas approximation

[11] a heavy ion is a degenerated gas of nucleons at $T = 0$ with a baryon density

$$n_B = \frac{A}{\dfrac{4\pi}{3} r_A^3} = \frac{3}{4\pi}\,\frac{1}{r_N^3} = 0.14\,{\rm fm}^{-3} \tag{2.1}$$

coinciding with the nuclear matter density $n_{\rm N}$ [11], where $r_A = r_N\,A^{1/3}$ is the radius of a heavy ion, and $r_N = 1.2\,{\rm fm}$ [11,12].

Suppose that all baryon degrees of freedom are converted into quark degrees of freedom and quarks are massless. In this case we should get a degenerated Fermi gas of free quarks or more generally a degenerated free quark–gluon system, where all gluon and anti–quark degrees of freedom are died out. Heating this quark–gluon system up to a temperature T we should arrive at a thermalized system of quarks, anti–quarks and gluons confined in the finite volume of a heavy ion $(4\pi/3) r_N^3\,A$.

In the low–temperature limit $T \to 0$ such a conversion of baryon degrees of freedom into quark ones requiring to have a system of free quarks can be understood qualitatively, for example, within a naive non–relativistic quark model, where baryons are slightly bound three–quark states. These three valence quarks, the constituent quarks, can be considered as current quark excitations above a quark condensate produced by a cloud of current $q\bar{q}$ pairs due to spontaneous breaking of chiral symmetry.

In terms of the light quark and anti–quark distribution functions Eq.(1.1) a baryon density of a thermalized quark–gluon system at a temperature T is given by [1,4]

$$\begin{aligned} n_B(T) &= \frac{1}{3} \times 2 \times 2 \times N_C \times [n_q(T) - n_{\bar{q}}(T)] = \\ &= \frac{4}{3}\,N_C \int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{e^{-\nu(T) + p/T} + 1} - \frac{1}{e^{\nu(T) + p/T} + 1}\right]. \end{aligned} \tag{2.2}$$

The factor $(1/3) \times 2 \times 2 \times N_C$ stands for the product of (*baryon charge*)$\times$ (*number of light quark flavour degrees of freedom*)$\times$(*number of spin degrees of freedom*)$\times$(*number of quark colour degrees of freedom*). The integration over the momentum $\vec{p}$ gives one [1]

$$n_B(T) = \frac{2}{9}\,N_C\left[\nu(T) + \frac{\nu^3(T)}{\pi^2}\right] T^3. \tag{2.3}$$

At zero temperature $T = 0$ we get

$$\mu_0 = \left(\frac{3\pi^2}{2}\right)^{1/3} n_{\rm B}^{1/3} = 250\,{\rm MeV}, \tag{2.4}$$

where $\mu_0 = \mu(0)$ is a chemical potential at zero temperature, and $n_{\rm B}(0) = n_{\rm B}$ defined by Eq.(2.1). We have set $n_{\rm B}(0) = n_{\rm B}$, where $n_{\rm B} = 0.14\,{\rm fm}^{-3}$ is given by Eq.(2.1), since in the fixed spatial volume due to a conservation of a baryon number the baryon density of nucleons should be equal to the baryon density of quarks. Our result $\mu_0 = 250\,{\rm MeV}$ agrees good with the estimate $\mu_0 \sim 300\,{\rm MeV}$ [1].

The fluctuations of the quark baryon number $n_B(T)$ caused by the fluctuations of a temperature T, produced in the fixed spatial volume $(4\pi/3) r_N^3\,A$ of the heavy ion, should lead to the violation of the baryon number. As the baryon number is a good quantum number conserved for strong interactions, we impose the constraint

$$n_B(T) = n_B. \tag{2.5}$$

Form Eq.(2.5) we define the chemical potential $\mu(T)$ as a function of T:

$$\frac{\mu(T)}{\mu_0} = \left[\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4\pi^6}{27}\left(\frac{T}{\mu_0}\right)^6}\,\right]^{1/3} - \left[-\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4\pi^6}{27}\left(\frac{T}{\mu_0}\right)^6}\,\right]^{1/3}. \tag{2.6}$$

The dependence of the chemical potential on a temperature T given by Eq.(2.6) governs the conservation of a baryon number for the thermalized quark–gluon system confined in the fixed volume $V = (4\pi/3)\,r^3_N\,A$, when a temperature changes itself.

In the low–temperature limit $T \to 0$ we get

$$\mu(T) = \mu_0\left[1 - \frac{\pi^2}{3}\frac{T^2}{\mu^2_0} + O\Big(T^6\Big)\right]. \tag{2.7}$$

The T–dependence of a chemical potential given by Eq.(2.7) differs by a factor 1/4 from the low–temperature behaviour of a chemical potential of a thermalized electron gas [13].

In the high–temperature limit $T \to \infty$ a chemical potential $\mu(T)$ defined by Eq.(2.7) drops like T^{-2} [1]:

$$\mu(T) = \frac{\mu^3_0}{\pi^2}\frac{1}{T^2} + O\Big(T^{-7}\Big). \tag{2.8}$$

A chemical potential drops very swiftly when a temperature increases. Indeed, at $T = 160\,\mathrm{MeV}$ we obtain $\mu(T) \simeq \mu_0/4$, while at $T = \mu_0$ a value of a chemical potential makes up about tenth part of μ_0, i.e. $\mu(T) \simeq \mu_0/10$. This implies that at very high temperatures the function $\nu(T) = \mu(T)/T$ becomes small and the contribution of a chemical potential of light quarks and anti–quarks can be taken into account perturbatively. This assumes in particular that at temperatures $T \ge \mu_0 = 250\,\mathrm{MeV}$ the number of light anti–quarks will not be suppressed by a chemical potential relative to the number of light quarks.

Thereby, at very high temperatures the ratio $R_{K^+K^-}(q,T)$ should tend to unity. Hence, the temperature dependence of a chemical potential given by Eq.(2.7) assumes that hadrons produced by the quark–gluon system through the QGP phase at temperatures $T > \mu_0 = 250\,\mathrm{MeV}$ can be hardly distinguished from the hadrons procreated by a quark–gluon system escaped from the QGP phase.

However, for intermediate temperatures $T = 160 \div 200\,\mathrm{MeV}$ the ratio $R_{K^+K^-}(q,T)$ would differ from the unity. Indeed, the rough estimate gives one

$$R_{K^+K^-}(q,T) \sim e^{2\nu(T)} = 2.14 \div 1.48\ >\ 1. \tag{2.9}$$

Hence, the analysis of the ratio $R_{K^+K^-}(q,T)$ can be still a good criterion for the signal of a thermalized QGP phase realized in ultra–relativistic heavy–ion collisions.

For the rough estimate of the ratio $R_{K^+\pi^+}(q,T)$ we obtain

$$R_{K^+\pi^+}(q,T) \sim \frac{V_K}{V_\pi}\ \times\ e^{\nu(T)} = 0.114 \div 0.059. \tag{2.10}$$

The estimates Eq.(2.9) and Eq.(2.10) are in qualitative agreement with the experimental data on the central ultra–relativistic Pb–Pb at 158 GeV per nucleon collisions by NA49 and NA44 Collaborations and the data on proton–nucleus and nucleus–nucleus collisions at 200 GeV per nucleon by NA35 Collaboration: $R^{\rm exp}_{K^+K^-} = 1.80 \pm 0.10$ [14–17] and $R^{\rm exp}_{K^+\pi^+} = 0.137 \pm 0.008$ [17].

3 Thermalized quark–gluon plasma

In average the QGP phase exists only for the interim of order of $\tau_{\rm QGP} = (6\div 15)\,{\rm fm/c}$. Therefore, the thermalization of this system should occur at times $\tau_{\rm th}$ much less than $\tau_{\rm QGP}$, i.e. $\tau_{\rm QGP} \gg \tau_{\rm th}$. This can be fulfilled in a very dense matter. Thereby, in order to be convinced that a thermalized QGP can be realized in ultra–relativistic heavy–ion collisions we have to calculate $n(T)$, a particle density of the QGP at a temperature T containing the contributions of quarks, anti–quarks and gluons, and to compare the value of $n(T)$ with the nuclear matter density $n_{\rm N} = 0.14\,{\rm fm}^{-3}$.

A particle density of a thermalized QGP is determined by [1,4]

$$\begin{aligned} n(T) &= n_g(T) + n_q(T) + n_{\bar q}(T) = 2(N^2_C - 1)\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{e^{p/T} - 1}\\ &+ 4\,N_C\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{e^{-\nu(T) + p/T} + 1} + 4\,N_C\int \frac{d^3p}{(2\pi)^3}\,\frac{1}{e^{\nu(T) + p/T} + 1}. \end{aligned}\tag{3.1}$$

Integrating over momenta we obtain

$$n(T) = T^3\,\frac{4N_C}{\pi^2}\,\Bigg[\Bigg(\frac{N^2_C - 1}{4N_C} + \frac{1}{2}\Bigg)\,\zeta(3) + \nu^2(T)\,\ell n\,2 + \frac{1}{6}\,\nu^3(T) - \int^{\nu(T)}_0 dx\,\frac{(\nu(T) - x)^2}{e^x + 1}\Bigg],\tag{3.2}$$

where $\zeta(3) = 1.202$ is a Riemann zeta–function defined by [18]

$$\zeta(s) = \frac{1}{\Gamma(s)}\int^{\infty}_0 dx\,\frac{x^{s-1}}{e^x - 1} = \frac{1}{(1 - 2^{1-s})}\,\frac{1}{\Gamma(s)}\int^{\infty}_0 dx\,\frac{x^{s-1}}{e^x + 1}.\tag{3.3}$$

At $N_C = 3$ the particle density $n(T)$ amounts to

$$n(T) = T^3\,\frac{12}{\pi^2}\,\Bigg[\frac{7}{6}\,\zeta(3) + \nu^2(T)\,\ell n\,2 + \frac{1}{6}\,\nu^3(T) - \int^{\nu(T)}_0 dx\,\frac{(\nu(T) - x)^2}{e^x + 1}\Bigg].\tag{3.4}$$

For temperatures $T \ge 160\,{\rm MeV}$ we can neglect the contribution of two last terms and with accuracy better than 1% the particle density is equal to

$$n(T) = T^3\,\frac{12}{\pi^2}\,\Bigg[\frac{7}{6}\,\zeta(3) + \nu^2(T)\,\ell n\,2\Bigg].\tag{3.5}$$

Setting $T \ge 160\,{\rm MeV}$ we get the estimate $n(T) \ge 0.98\,{\rm fm}^{-3}$. This density is by a factor of order 7 larger compared with the nuclear matter density $n_{\rm N} = 0.14\,{\rm fm}^{-3}$.

Thus, if we consider quarks, anti–quarks and gluons of a thermalized QGP like rigid spheres with fixed radii, the average time of collisions between particles in the QGP is much less than $D/c = (6/\pi n(T)c^3)^{1/3} \simeq 1\,{\rm fm/c}$, i.e. $\tau_{coll} \ll 1\,{\rm fm/c}$. This implies that $\tau_{\rm th}$, the time of a thermalization of the QGP, should be of order $\tau_{\rm th} \le 1\,{\rm fm/c}$. This value is of order of magnitude less compared with the life time of the QGP, i.e. $\tau_{QGP} \ge (6\div 15)\,\tau_{\rm th}$. These estimates make admissible the application of a thermalized quark–gluon system to the description of the QGP phase.

For a thermalized QGP the energy density is determined by [1,4]:

$$\begin{aligned} \varepsilon(T) &= \varepsilon_g(T) + \varepsilon_q(T) + \varepsilon_{\bar q}(T) = 2(N^2_C - 1)\int \frac{d^3p}{(2\pi)^3}\,\frac{p}{e^{p/T} - 1}\\ &+ 4\,N_C\int \frac{d^3p}{(2\pi)^3}\,\frac{p}{e^{-\nu(T) + p/T} + 1} + 4\,N_C\int \frac{d^3p}{(2\pi)^3}\,\frac{p}{e^{\nu(T) + p/T} + 1}. \end{aligned}\tag{3.6}$$

Integrating over momenta we get [1,4]:

$$\varepsilon(T) = T^4\, N_C \left[\frac{N^2_C - 1}{N_C}\,\frac{\pi^2}{15} + \frac{7\pi^2}{30} + \nu^2(T) + \frac{1}{2\pi^2}\,\nu^4(T)\right]. \tag{3.7}$$

At $N_C = 3$ the energy density of a thermalized QGP amounts to

$$\varepsilon(T) = T^4 \left[\frac{37\pi^2}{30} + 3\,\nu^2(T) + \frac{3}{2\pi^2}\,\nu^4(T)\right]. \tag{3.8}$$

Setting $T \geq 160\,\mathrm{MeV}$ we estimate $\varepsilon(T) \geq 1.08\,\mathrm{GeV/fm^3}$. Such values of the energy density are enough for the existence of the QGP [1,4]. Thus, our estimate is on favour of the existence of a thermalized QGP phase of the quark–gluon system in ultra–relativistic heavy–ion collisions. Therefore, we can proceed to the evaluation of the multiplicities of the $K^{\pm}$– and $\pi^{\pm}$–meson production caused by a hadronization of the QGP.

4 Multiplicities of the $K^{\pm}$–meson production

The evaluation of the multiplicities of the $K^{\pm}$–meson production caused by a hadronization of the QGP we suggest to perform by making use of Eq.(1.4). Since in our approach to the QGP quarks, anti–quarks and gluons are described as free Fermi and Bose gases and the quark–gluon interactions are practically switched off, we follow a simple coalescence approach to the hadronization [4,6]. Thus, the multiplicities of the $K^{\pm}$–meson production will be evaluated through the formulae (see Eq.(1.4))

$$\begin{aligned} N_{K^+}(\vec{q}, T) &= 3\,V_K \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{e^{-\nu(T) + |\vec{p} - \vec{q}\,|/T} + 1}\,\frac{1}{e^{\sqrt{\vec{p}^{\,2} + m^2_s}/T} + 1},\\ N_{K^-}(\vec{q}, T) &= 3\,V_K \int \frac{d^3p}{(2\pi)^3}\,\frac{1}{e^{\nu(T) + |\vec{p} - \vec{q}\,|/T} + 1}\,\frac{1}{e^{\sqrt{\vec{p}^{\,2} + m^2_s}/T} + 1}, \end{aligned} \tag{4.1}$$

where we have set $N_C = 3$.

We show below that the multiplicities given by Eq.(4.1) are the functions of $\lambda = e^{\nu(T)}$, $\lambda_s = e^{m_s/T}$ and $\lambda_K = e^{q/T}$:

$$\begin{aligned} N_{K^+}(\vec{q}, T) &= N_{K^+}(\lambda, \lambda_K, \lambda_s),\\ N_{K^-}(\vec{q}, T) &= N_{K^+}(\lambda^{-1}, \lambda_K, \lambda_s), \end{aligned} \tag{4.2}$$

where a 3–momentum q is related to a rapidity y, a transversal momentum $\vec{q}_{\perp}$ and the mass $M_K = 500\,\mathrm{MeV}$ of the $K^{\pm}$ mesons as $q = \sqrt{M^2_K \mathrm{sh}^2 y + \vec{q}^{\,2}_{\perp}\mathrm{ch}^2 y}$.

Integrating over the directions of the momentum $\vec{p}$ we obtain

$$\begin{aligned} N_{K^+}(\lambda, \lambda_K, \lambda_s) &= \frac{3m^2_s T V_K}{4\pi^2 \ell n\,\lambda_K}\int\limits_0^{\varphi(q)} d\varphi\,\frac{\mathrm{sh}\varphi\,\mathrm{ch}\varphi}{1 + \lambda_s^{\mathrm{ch}\varphi}}\\ &\times \Bigg[\int\limits_0^{\lambda} dz\,\frac{\ell n\Big(1 + z\,\lambda^{-1}_K\,\lambda_s^{\mathrm{sh}\varphi}\Big)}{z} - \int\limits_0^{\lambda} dz\,\frac{\ell n\Big(1 + z\,\lambda^{-1}_K\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}\\ &+ (\ell n\,\lambda_K - \ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1 + \lambda\,\lambda^{-1}_K\,\lambda_s^{\mathrm{sh}\varphi}\Big) \end{aligned}$$

$$-(\ell n\,\lambda_K+\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)\Big]$$

$$+\frac{3m_s^2TV_K}{4\pi^2\ell n\,\lambda_K}\int\limits_{\varphi(q)}^{\infty}d\varphi\,\frac{\mathrm{sh}\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}$$

$$\times\Big[\int\limits_0^{\lambda}dz\,\frac{\ell n\Big(1+z\,\lambda_K\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}-\int\limits_0^{\lambda}dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}$$

$$-(\ell n\,\lambda_K-\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K\,\lambda_s^{-\mathrm{sh}\varphi}\Big)$$

$$-(\ell n\,\lambda_K+\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)\Big], \qquad (4.3)$$

where we have also changed a variable $p=m_s\,\mathrm{sh}\varphi$ and denoted

$$\varphi(q)=\ell n\bigg(\frac{q}{m_s}+\sqrt{1+\frac{q^2}{m_s^2}}\bigg). \qquad (4.4)$$

For the integration over directions $\vec{p}$ we have used the formula

$$\int\frac{dx\,x}{\lambda^{-1}\,e^{x}+1}=-x\,\ell n\Big(1+\lambda\,e^{-x}\Big)-\int\limits_0^{\lambda}dz\,\frac{\ell n\Big(1+z\,e^{-x}\Big)}{z}. \qquad (4.5)$$

Using Eq.(4.2) we are able to define the ratio of the multiplicities of the $K^{\pm}$–meson production as follows

$$R_{K^+K^-}(q,T)=\frac{N_{K^+}(\lambda,\lambda_K,\lambda_s)}{N_{K^+}(\lambda^{-1},\lambda_K,\lambda_s)}=$$

$$=\bigg\{\int\limits_0^{\varphi(q)}d\varphi\,\frac{\mathrm{sh}\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}$$

$$\times\Big[\int\limits_0^{\lambda}dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{\mathrm{sh}\varphi}\Big)}{z}-\int\limits_0^{\lambda}dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}$$

$$+(\ell n\,\lambda_K-\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K^{-1}\,\lambda_s^{\mathrm{sh}\varphi}\Big)$$

$$-(\ell n\,\lambda_K+\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)\Big]$$

$$+\int\limits_{\varphi(q)}^{\infty}d\varphi\,\frac{\mathrm{sh}\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}$$

$$\times\Big[\int\limits_0^{\lambda}dz\,\frac{\ell n\Big(1+z\,\lambda_K\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}-\int\limits_0^{\lambda}dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}$$

$$-(\ell n\,\lambda_K-\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K\,\lambda_s^{-\mathrm{sh}\varphi}\Big)$$

$$-(\ell n\,\lambda_K+\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)\Big]\bigg\}$$

$$
\begin{aligned}
&\times\Big\{\int\limits_0^{\varphi(q)} d\varphi\,\frac{\mathrm{sh}\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}\\
&\times\Big[\int\limits_0^{1/\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{\mathrm{sh}\varphi}\Big)}{z}-\int\limits_0^{1/\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}\\
&+(\ell n\,\lambda_K-\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda^{-1}\,\lambda_K^{-1}\,\lambda_s^{\mathrm{sh}\varphi}\Big)\\
&-(\ell n\,\lambda_K+\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda^{-1}\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)\Big]\\
&+\int\limits_{\varphi(q)}^{\infty} d\varphi\,\frac{\mathrm{sh}\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}\\
&\times\Big[\int\limits_0^{1/\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_K\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}-\int\limits_0^{1/\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}\\
&-(\ell n\,\lambda_K-\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda^{-1}\,\lambda_K\,\lambda_s^{-\mathrm{sh}\varphi}\Big)\\
&-(\ell n\,\lambda_K+\ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda^{-1}\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)\Big]\Big\}^{-1}. \qquad (4.6)
\end{aligned}
$$

Thus, due to λ, λ_K and λ_s the ratio of the multiplicities of the $K^{\pm}$–meson production depends on the 3–momenta of the $K^{\pm}$ mesons, i.e. a rapidity y and a transversal momentum $|\vec{q}_{\perp}|$, and a temperature T.

For high momenta $q\to\infty$, i.e. high rapidities $y\to\infty$ or high transversal momenta $|\vec{q}_{\perp}|\to\infty$, the multiplicities of the $K^{\pm}$–meson production can be substantially simplified. Indeed, at the limit $q\to\infty$ or the limit $\lambda_K\to\infty$ the contribution of the integrals over the region $\varphi(q)\le\varphi<\infty$ can be neglected relative to the contribution of the integrals over the region $0\le\varphi\le\varphi(q)$. Keeping then only the leading terms in large λ_K expansion we arrive at the expressions

$$
N_{K^{\pm}}(\lambda,\lambda_K,\lambda_s)=\frac{3T^3V_K}{4\pi^2}\,\lambda^{\pm1}\,\lambda_K^{-1}=\frac{3T^2V_K}{4\pi^2}\,e^{\pm\nu(T)}\,e^{-q/T}, \qquad (4.7)
$$

where the factor $e^{-q/T}$ testifies a hadronization of the thermalized QGP at a temperature T into a thermalized ultra–relativistic $K^{\pm}$–meson gas at a temperature T.

Taking then the ratio $R_{K^+K^-}(q,T)$ at the limit $q\to\infty$ we obtain

$$
\lim_{q\to\infty}R_{K^+K^-}(q,T)=R_{K^+K^-}(\infty,T)=\lambda^2. \qquad (4.8)
$$

Hence, in ultra–relativistic heavy–ion collisions going through the intermediate QGP phase described by the free thermalized quark–gluon gas at a temperature T the multiplicities of the $K^{\pm}$–meson production as functions of λ_K decrease like $\ell n\,\lambda_K/\lambda_K$. In turn the ratio of the multiplicities $R_{K^+K^-}(q,T)$ does not depend on the momenta of the $K^{\pm}$–mesons and becomes defined only by a chemical potential of the light quarks. We show below that the prediction Eq.(4.8) agrees good with the experimental data on central ultra–relativistic Pb–Pb collisions at 158 GeV per nucleon in the temperature interval $160\,\mathrm{MeV}\le T\le 200\,\mathrm{MeV}$.

The ratio $R_{K^+K^-}(\infty,T)$ given by Eq.(4.8) differs from the result obtained by Koch, Müller and Rafelski (see Eq.(6.29) of Ref.[4b]) by a factor $\lambda_s^2=\exp(2\mu_s/kT)$, the squared fugacity of strange quarks, where μ_s is a chemical potential of strange quarks. In the case of chemical equilibrium which we follow in our approach $\mu_s=0$ and $\lambda_s=\lambda_{\bar{s}}^{-1}=1$.

In the limit $q \to 0$ the multiplicities of the $K^{\pm}$–meson production behave like

$$N_{K^{\pm}}(\lambda, 1, \lambda_s) \; = \; \frac{3m_s^3 V_K}{2\pi^2}\int\limits_0^{\infty} d\varphi\,\frac{\mathrm{sh}^2\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}\,\frac{1}{1+\lambda^{\mp 1}\,\lambda_s^{\mathrm{sh}\varphi}}. \tag{4.9}$$

It is easy to see that the main contribution to $N_{K^{\pm}}(\lambda, 1, \lambda_s)$ comes from the region $\varphi(q) \le \varphi < \infty$. Thus, in the limit $q \to 0$ the ratio of the multiplicities amounts to

$$R_{K^+K^-}(0,T) = \Bigg[\int\limits_0^{\infty} d\varphi\,\frac{\mathrm{sh}^2\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}\,\frac{1}{1+\lambda^{-1}\,\lambda_s^{\mathrm{sh}\varphi}}\Bigg]\Bigg[\int\limits_0^{\infty} d\varphi\,\frac{\mathrm{sh}^2\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}\,\frac{1}{1+\lambda\,\lambda_s^{\mathrm{sh}\varphi}}\Bigg]^{-1}. \tag{4.10}$$

By varying the momenta of the $K^{\pm}$ mesons over the region $0 \le q < \infty$ one should get the ratio $R_{K^+K^-}(q,T)$ changing between the values defined by Eq.(4.10) and Eq.(4.8).

5 Multiplicity of the $\pi^{\pm}$–meson production

In our approach the multiplicities $N_{\pi^+}(\vec{q},T)$ and $N_{\pi^-}(\vec{q},T)$ of the production of the π^+ and π^- mesons are equal and defined by Eq.(1.5). Integrating over directions of the momentum $\vec{p}$ we arrive at the expression depending only on λ and $\lambda_\pi = e^{q/T}$:

$$\begin{aligned}
N_{\pi^+}(\lambda,\lambda_\pi) \; &= \; \frac{3V_\pi T^3}{4\pi^2\,\ell n\lambda_\pi}\int\limits_0^{\lambda_\pi}\frac{dx\,x}{\lambda\,e^{x}+1}\Bigg[\int\limits_0^{\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_\pi^{-1}\,e^{x}\Big)}{z} - \int\limits_0^{\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_\pi^{-1}\,e^{-x}\Big)}{z}\\
&\quad +(\ell n\,\lambda_\pi - x)\,\ell n\Big(1+\lambda\,\lambda_\pi^{-1}\,e^{x}\Big) - (\ell n\,\lambda_\pi + x)\,\ell n\Big(1+\lambda\,\lambda_\pi^{-1}\,e^{-x}\Big)\Bigg]\\
&+ \; \frac{3VT^3}{4\pi^2\,\ell n\lambda_\pi}\int\limits_{\lambda_\pi}^{\infty}\frac{dx\,x}{\lambda\,e^{x}+1}\Bigg[\int\limits_0^{\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_\pi\,e^{-x}\Big)}{z} - \int\limits_0^{\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_\pi^{-1}\,e^{-x}\Big)}{z}\\
&\quad -(\ell n\,\lambda_\pi + x)\,\ell n\Big(1+\lambda\,\lambda_\pi^{-1}\,e^{-x}\Big) - (\ell n\,\lambda_\pi - x)\,\ell n\Big(1+\lambda\,\lambda_\pi^{-1}\,e^{-x}\Big)\Bigg].
\end{aligned} \tag{5.1}$$

The ratio of the production of the π^+ mesons with respect to the K^+ mesons is determined by the ratio $R_{K^+\pi^+}(q,T)$ which reads

$$\begin{aligned}
&R_{K^+\pi^+}(q,T) = \frac{N_{K^+}(\lambda,\lambda_K,\lambda_s)}{N_{\pi^+}(\lambda,\lambda_\pi)} = \frac{m_s^2}{T^2}\,\frac{V_K}{V_\pi}\Bigg\{\int\limits_0^{\varphi(q)} d\varphi\,\frac{\mathrm{sh}\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}\\
&\times\Bigg[\int\limits_0^{\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{\mathrm{sh}\varphi}\Big)}{z} - \int\limits_0^{\lambda} dz\,\frac{\ell n\Big(1+z\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)}{z}\\
&+(\ell n\,\lambda_K - \ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K^{-1}\,\lambda_s^{\mathrm{sh}\varphi}\Big)\\
&-(\ell n\,\lambda_K + \ell n\lambda_s\,\mathrm{sh}\varphi)\,\ell n\Big(1+\lambda\,\lambda_K^{-1}\,\lambda_s^{-\mathrm{sh}\varphi}\Big)\Bigg]\\
&+\int\limits_{\varphi(q)}^{\infty} d\varphi\,\frac{\mathrm{sh}\varphi\,\mathrm{ch}\varphi}{1+\lambda_s^{\mathrm{ch}\varphi}}
\end{aligned}$$

$$
\times \Bigg[\int\limits_0^{\lambda} dz\, \frac{\ell n\Big(1 + z\,\lambda_K\,\lambda_s^{-{\rm sh}\varphi}\Big)}{z} - \int\limits_0^{\lambda} dz\, \frac{\ell n\Big(1 + z\,\lambda_K^{-1}\,\lambda_s^{-{\rm sh}\varphi}\Big)}{z}
$$

$$
-(\ell n\,\lambda_K - \ell n\lambda_s\,{\rm sh}\varphi)\,\ell n\Big(1 + \lambda\,\lambda_K\,\lambda_s^{-{\rm sh}\varphi}\Big)
$$

$$
-(\ell n\,\lambda_K + \ell n\lambda_s\,{\rm sh}\varphi)\,\ell n\Big(1 + \lambda\,\lambda_K^{-1}\,\lambda_s^{-{\rm sh}\varphi}\Big)\Bigg]\Bigg\}
$$

$$
\times \Bigg\{ \int\limits_0^{\ell n\,\lambda_\pi} \frac{dx\,x}{\lambda\,e^{x} + 1}\Bigg[\int\limits_0^{\lambda} dz\, \frac{\ell n\Big(1 + z\,\lambda_\pi^{-1}\,e^{x}\Big)}{z} - \int\limits_0^{\lambda} dz\, \frac{\ell n\Big(1 + z\,\lambda_\pi^{-1}\,e^{-x}\Big)}{z}
$$

$$
+(\ell n\,\lambda_\pi - x)\,\ell n\Big(1 + \lambda\,\lambda_\pi^{-1}\,e^{x}\Big) - (\ell n\,\lambda_\pi + x)\,\ell n\Big(1 + \lambda\,\lambda_\pi^{-1}\,e^{-x}\Big)\Bigg]
$$

$$
+ \int\limits_{\ell n\,\lambda_\pi}^{\infty} \frac{dx\,x}{\lambda\,e^{x} + 1}\Bigg[\int\limits_0^{\lambda} dz\, \frac{\ell n\Big(1 + z\,\lambda_\pi\,e^{-x}\Big)}{z} - \int\limits_0^{\lambda} dz\, \frac{\ell n\Big(1 + z\,\lambda_\pi^{-1}\,e^{-x}\Big)}{z}
$$

$$
-(\ell n\,\lambda_\pi - x)\,\ell n\Big(1 + \lambda\,\lambda_\pi\,e^{-x}\Big) - (\ell n\,\lambda_\pi + x)\,\ell n\Big(1 + \lambda\,\lambda_\pi^{-1}\,e^{-x}\Big)\Bigg]\Bigg\}^{-1}. \qquad (5.2)
$$

For high momenta $q \to \infty$ the multiplicity of the $\pi^{\pm}$–meson production behave like the multiplicity of the $K^{\pm}$–meson production

$$
N_{\pi^{\pm}}(\lambda, \lambda_\pi) = \frac{3T^3 V_\pi}{4\pi^2}\,\lambda_\pi^{-1} = \frac{3T^3 V_\pi}{4\pi^2}\,e^{-q/T}, \qquad (5.3)
$$

where the factor $e^{-q/T}$ testifies a hadronization of the thermalized QGP at a temperature T into a thermalized ultra–relativistic $\pi^{\pm}$–meson gas at a temperature T.

Taking into account Eq.(4.7) we obtain the ratio $R_{K^+\pi^+}(q,T)$ at the limit $q \to \infty$:

$$
\lim_{q\to\infty} R_{K^+\pi^+}(q,T) = R_{K^+\pi^+}(\infty,T) = \frac{V_K}{V_\pi}\,e^{\nu(T)}. \qquad (5.4)
$$

Below we show that this relation agrees good with the experimental data on nucleus–nucleus ultra–relativistic collisions at 200 GeV per nucleon (NA35 Collaboration).

It is remarkable that in the ratio a parameter C is canceled and the ratio $R_{K^+\pi^+}(q,T)$ is determined by good established parameters of the K^+ and π^+ mesons such as the masses and the leptonic coupling constants.

6 Numerical analysis

The numerical analysis of the ratio $R_{K^+K^-}(q,T)$ given by Eq(4.6) displays that it slightly varies around the value $R_{K^+K^-}(\infty,T) = \lambda^2$, when 3–momenta of the $K^{\pm}$ mesons take values from the region $0 \le q < 10^3$ GeV, and it satisfies the relation Eq.(4.8) at $q \ge 10^3$ GeV.

The ratio Eq.(4.6) depends also smoothly on a temperature ranging over the region $160\,{\rm MeV} \le T < 200\,{\rm MeV}$. The numerical data read

$$
\begin{aligned}
R_{K^+K^-}(q, T = 160) &= 2.14^{+0.13}_{-0.30},\\
R_{K^+K^-}(q, T = 175) &= 1.80^{+0.04}_{-0.18},\\
R_{K^+K^-}(q, T = 190) &= 1.58^{+0.02}_{-0.13},
\end{aligned} \qquad (6.1)
$$

where the upper and the lower values correspond to the maximum and the minimum of the ratio, respectively.

When matching the theoretical values Eq.(6.1) with the experimental data on central ultra–relativistic PB–Pb collisions at 158/,GeV per nucleon [14–17]:

$$R^{\rm exp}_{K^+K^-} = 1.80 \pm 0.10, \tag{6.2}$$

one can see that our approach describes good the experimental data on the production of the $K^{\pm}$–mesons at the temperature $T = 175\,{\rm MeV}$.

Since in the case of a hadronization of a quark–gluon system escaped from the QGP phase we have obtained the ratio $R_{K^+K^-}$ independent on the momentum of the $K^{\pm}$–mesons and equal to $R_{K^+K^-} = 1.10 \pm 0.01$, the numerical estimates Eq.(6.1) evidently testify that the intermediate state in ultra–relativistic heavy–ion collisions should run via the QGP phase with a handsome probability.

For the entire region of the 3–momenta $0 \le q < \infty$ the ratio $R_{K^+\pi^+}(q,T)$ of the multiplicities of the K^+ and π^+ meson production varies very smoothly and is given by

$$\begin{aligned} R_{K^+\pi^+}(q, T = 160) &= 0.144 \pm 0.017,\\ R_{K^+\pi^+}(q, T = 175) &= 0.134 \pm 0.014,\\ R_{K^+\pi^+}(q, T = 190) &= 0.128 \pm 0.011, \end{aligned} \tag{6.3}$$

where $\pm\Delta$ gives the minimum and the maximum values of the ratio. When matching the theoretical values of the ratio Eq.(6.3) with the experimental data, given by NA35 Collaboration on proton–nucleus and nucleus–nucleus collisions at $200\,{\rm GeV}$ per nucleon [17]

$$R^{\rm exp}_{K^+\pi^+} = 0.137 \pm 0.008, \tag{6.4}$$

one can see that the best agreement we get again for $T = 175\,{\rm MeV}$.

7 Conclusion

The main point of our approach to the description of the QGP phase as a thermalized quark–gluon system is in the determination of a chemical potential of light quarks $\mu(T)$. We have defined $\mu(T)$ as a function of a temperature T and calculated its value at $T = 0$ in terms of the baryon density of nucleons $n_{\rm B}$, which coincides with the nuclear matter density $n_{\rm B} = n_{\rm N} = 0.14\,{\rm fm}^{-3}$. Our result for the chemical potential at zero temperature $\mu_0 = 250\,{\rm MeV}$ agrees good with the estimate $\mu_0 \sim 300\,{\rm MeV}$ [1].

For the evaluation of multiplicities of the $K^{\pm}$–meson production we have followed a simple coalescence model defining multiplicities in terms of geometrical probabilities for pairs $u\bar{s}$ (or $\bar{u}s$) to be at a spatial volume V_K, where probabilities quarks and anti–quarks are given by Fermi distribution functions at a temperature T. We have shown that in the ultra–relativistic limit these multiplicities acquire the shape of the Maxwell–Boltzmann distribution functions testifying the existence of a thermalized $K^{\pm}$–meson gas at temperature T in the hadronic phase of the thermalized QGP.

The ratio of these multiplicities $R_{K^+K^-}(q,T)$ has turned out to be a smooth function on 3–momenta q of the $K^{\pm}$ mesons in the entire region $0 \le q \le \infty$ varying slightly around the values $R_{K^+K^-}(\infty, T) = e^{2\mu(T)/T}$. At $T = 175\,{\rm MeV}$ we have found a value $R_{K^+K^-}(q, T = 175) = 1.80^{+0.04}_{-0.18}$ agreeing good with the experimental data on central ultra–relativistic Pb–Pb collisions at 158 GeV per nucleon (NA49 and NA44 Collaborations) and ultra–relativistic nucleus–nucleus collisions at 200 GeV per nucleon (NA35 Collaboration) [14–17]: $R^{\rm exp}_{K^+K^-} =$

1.80 ± 0.10. Therewith, the thermodynamical parameters of our fit of experimental data $T = 175\,\mathrm{MeV}$ and $\mu(T = 175) = 51\,\mathrm{MeV}$ are in qualitative agreement with the experimental ones [15]: $T \sim 170\,\mathrm{MeV}$ and $\mu \sim 85\,\mathrm{MeV}$.

The ratio of the multiplicities of the $K^{\pm}$–meson production in the thermalized QGP has been calculated as a function of a chemical potential of light quarks and a temperature by Koch, Müller and Rafelski [4b]. When matching our result given for the ratio $R_{K^+K^-}(q,T)$ by Eqs.(4.6) and (6.1) with the result obtained by Koch, Müller and Rafelski (see Eq.(6.29) and Fig. 6.7 of Ref.[4b]) we argue that in the QGP phase of QCD (i) strange quark and anti–quarks are in the equilibrium state that provides a vanishing value of their chemical potential, (ii) the ratio of multiplicities is a smooth function of 3–momenta of $K^{\pm}$ mesons, (iii) the most important values of a temperature are higher than $T = 160\,\mathrm{MeV}$ and (iv) a chemical potential of light quarks is a swiftly dropping with T function. Then, we would like to emphasize that unlike the approach given by Koch, Müller and Rafelski Ref.[4b] in our model there are no free parameters. This makes the obtained results much more sensitive when compared with the experimental data, and the agreement with the experimental data, if reached, should signify a correct mechanism of the QGP. Indeed, the ratio $R_{K^+K^-}(q,T)$ depending only on a temperature can be unambiguously compared with the experimental data by varying only a temperature T in reasonable limits.

The ratio $R_{K^+\pi^+}(q,T)$ of the multiplicities of the K^+ and π^+–meson production has been found as a smooth function of the 3–momenta $0 < q < \infty$. The absolute value of the ratio $R_{K^+\pi^+}(q,T)$ depends on the parameter of the model V_K/V_π. The ratio V_K/V_π is fixed in our approach in terms of good established parameters of the K^+ and π^+ mesons: $V_K/V_\pi = (F_\pi M_\pi/F_K M_K)^{3/2} = 0.109$. This gives the ratio $R_{K^+\pi^+}(q,T)$ which agrees good with experimental data given by NA35 Collaboration [17]: $R^{\rm exp}_{K^+\pi^+} = 0.137 \pm 0.008$. The best agreement we get at $T = 175\,\mathrm{MeV}$: $R_{K^+\pi^+}(q,T = 175) = 0.134 \pm 0.014$. At $T = 175\,\mathrm{MeV}$ the ratio $R_{K^+\pi^+}(q,T)$ changes from a minimal value $R_{K^+\pi^+}(q,T) = 0.120$ to a maximal one $R_{K^+\pi^+}(q,T) = 0.148$ with a 3–momentum ranging over the region $q \in [0,\infty)$.

First calculation of the ratio of the multiplicities of the K^+– and π^+–meson production in the thermalized QGP has been performed by Glendenning and Rafelski [19]. Unlike our approach in Ref.[19] for the description of the multiplicity of the production of π^+ mesons there has been used the distribution function corresponding to the consideration of the produced π^+ mesons as a thermalized Bose gas. To some extent this leads to a loss of the quark origin of π^+ mesons which is retained in our approach Eq.(1.5). When comparing the numerical value of the ratio $R_{K^+\pi^+}(q,T)$ obtained in our model with that calculated by Glendenning and Rafelski $K^+/\pi^+ \approx 0.3$ (see Fig. 2 of Ref.[19]) for $T = 160 \div 180\,\mathrm{MeV}$ we should testify that our result agrees better with the contemporary experimental data.

Such an agreement obtained for $R_{K^+K^-}(q,T)$ and $R_{K^+\pi^+}(q,T)$ seems to be rather interesting, since in our approach there is no free parameters save a temperature T. Indeed, a chemical potential at zero temperature $\mu_0 = 250\,\mathrm{MeV}$ is determined through the nuclear matter density $n_{\rm N} = 0.14\,\mathrm{fm}^{-3}$. Then, the mass of a current s–quark $m_s = 135\,\mathrm{MeV}$ is quoted by QCD at the normalization scale $1\,\mathrm{GeV}$ [8], and it has been successfully applied to numerous calculations of fine chiral structure of both light and heavy–light mesons [9,10]. This implies that our definition of a chemical potential $\mu(T)$ carried out through the requirement of a conservation of a heavy ion baryon number at any temperature T describes good a thermodynamical properties of the QGP in the form of a thermalized free Fermi–Bose gas of quarks, anti–quarks and gluons. Then, an arbitrary parameter C entering to the definition of the parameters V_K and V_π, the coalescence model parameters, has been canceled in the ratio V_K/V_π. As a result the ratio $R_{K^+\pi^+}(q,T)$ has been defined by good established parameters of K and π mesons: the masses $M_\pi = 140\,\mathrm{MeV}$ and $M_K = 500\,\mathrm{MeV}$, and the leptonic coupling constants $F_\pi = 131\,\mathrm{MeV}$ and $F_K = 160\,\mathrm{MeV}$.

In the case of a hadronization of a quark–gluon system escaped from the QGP phase we

have obtained the ratio $R_{K^+K^-}$ independent on the momentum of the $K^{\pm}$–mesons and equal to $R_{K^+K^-} = 1.10 \pm 0.01$. Therefore, the numerical estimates of the ratio $R_{K^+K^-}(q,T)$ carried out in out approach to the description of the QGP evidently testify that the intermediate state in ultra–relativistic heavy–ion collisions should run via the QGP phase with a handsome probability.

Our success in describing of the ratios of the $K^{\pm}$– and $\pi^{\pm}$–meson productions in ultra–relativistic heavy–ion collisions with a minimal number of free parameters should be supported by the description of the ratios of the strange baryon and anti–baryon production [20]: $R_{\bar{\Lambda}/\bar{\Sigma}\,\Lambda/\Sigma} = 0.133 \pm 0.007$, $R_{\bar{\Xi}\,\Xi} = 0.249 \pm 0.019$ and $R_{\bar{\Omega}\,\Omega} = 0.383 \pm 0.081$. The analysis of these experimental data in our approach are planned in our forthcoming publications.

When comparing our approach with others we would like to refer to the paper by Biró and Zimanýi [4a]. Indeed, our constraint $n_{\rm B}(T) = n_{\rm B}$ given by Eq.(2.5), allowing to determine a temperature dependence of a chemical potential, corresponds to some extent to the relation Eq.(2.18) of Ref.[4a]. This relation can be obtained from Eq.(2.5) by multiplying the r.h.s. of Eq.(2.5) by the factor $J\gamma_0$, where γ_0 is a Lorentz factor related to the laboratory energy of colliding heavy ions and J is a parameter of the approach [4a]. This gives $n_{\rm B}(T) = n_{\rm B}\,J\,\gamma_0$. According to Ref.[4a] the factor $J\,\gamma_0$ is always greater than unity $J\,\gamma_0 > 1$, whereas in our approach $J\gamma_0 = 1$. Unlike our approach, where all parameters are fixed, the parameter J is a free parameter of the model [4a].

8 Acknowledgement

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